Analysis of a Cavern Near the Flank of a Salt Dome

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ABSTRACT

The time-dependent behavior of an empty solution mined cavern in salt located near a lateral vertical boundary (flank) of its host salt dome is studied, the particular geometry considered inspired by an existing near-flank cavern in salt. A two-dimensional, plane-strain simplication of the interaction between cavern and flank is modeled using the finite element method. Only two materials are assumed involved: salt, interior to the dome, and shale, representing material outlying the dome. Creeping of salt is represented as very viscous fluid flow with an effective viscosity for salt defined by use of an empirical, time-hardening power law. The behavior of the material referred to as shale is simply assumed twice as viscous as salt under the same conditions. Material constants employed in the analysis were based on graphical triaxial extension data for salt available in the literature. The finite element method employed uses four-node quadrilateral elements and imposes a condition of approximate incompressibility by the use of a penalty function. Results obtained indicate a stress distribution showing little effect attributable to the proximity of the flank to the cavern.

INTRODUCTION

The purpose of this study is to develop and test a preliminary model of the interaction between a cavern in salt and a nearby lateral vertical boundary (flank) of the host salt dome. Focus is directed toward a particular cavern and flank geometry which was inspired by an existing solution-mined cavern located near a dome flank. The study originated as a demonstration model in a master's thesis relating to the implementation of a finite element code for modeling the slow, time-dependent deformation of rock salt (Gehle, 1980).

Data for the cavern describes it as roughly cylindrical in shape with a maximum radius of approximately 200 ft (61.0 m). Top and bottom of the cavern are located at 4125 ft (1257.3 m) and 4348 ft (1325.0 m), respectively. Based on seismic and surveying data, the minimum distance from the cavern wall to material outlying the dome is estimated to be about 250 ft (76.2 m). The dome flank near the cavern slopes gently inward from vertical. A schematic of the cavern and flank configuration is shown in Figure 1. Symmetry is implied across the exposed face containing the cavern.

The geometry depicted implies that the model be approached as a three-dimensional analysis task. However, a two-dimensional simplification can be had by neglecting cavern end effects, and assuming that the ratio of cavern depth to height is sufficiently large to approach plane

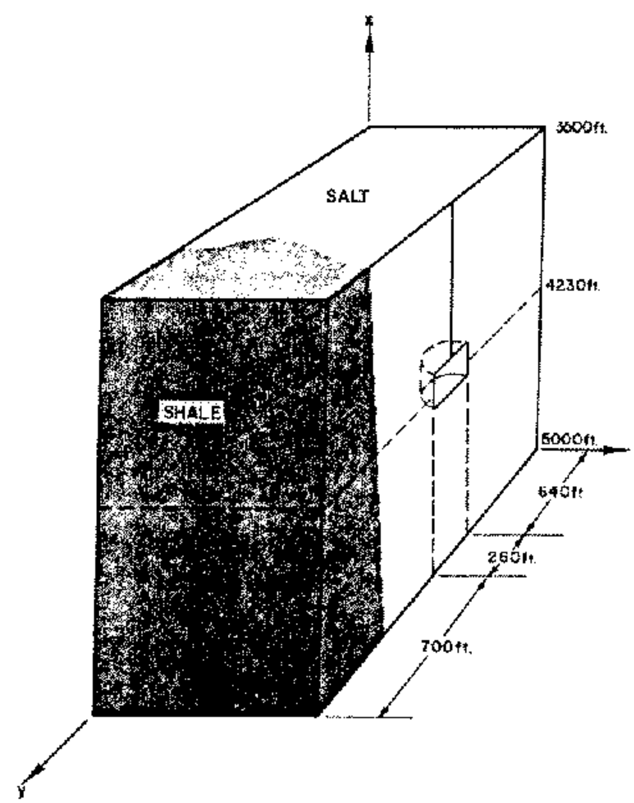

Figure 1. Schematic of a Cavern Near a Lateral Vertical Boundary of a Salt Dome

strain conditions at a horizontal plane passing through the cavern at mid-height. A plane-strain simplification was chosen as a first-approximation to the configuration in Figure 1. A further simplification is the assumption that only two materials are involved, rock salt, interior to the dome, and shale, representing materials outlying the dome.

MATERIAL REPRESENTATION

Rock salt under constant stress exhibits a deformation over time that resembles the flow of a very viscous fluid. Following this description the components of deviatoric stress Sij can be related to the components of strain-rate Eij by the constitutive expression:

$$\mathrm{Sij} = u\,\mathrm{Eij} \tag{1}$$

The proportionality term u plays the role of an effective viscosity. Definition of the viscosity allows selection of several of the material representations presently employed in the modeling of salt. The form taken here is after the work of others (Boresi, 1983; Thoms, *et al.*, 1972) and yields an empirical, time-hardening power law for the time-dependent response of rock salt to load. The effective viscosity is given by:

$$\mu = (K t^a J_2^b)^{-1} \tag{2}$$

in which t is time, J_2 is the second invariant of deviatoric stress, and K, a, b are material constants.

A fit of the employed material description to selected data from available graphical triaxial extension creep curves for salt (Boresi, 1983) is shown in Figure 2. In that figure points represent selected data and solid lines represent the calculated fit. For time in hours and stress in psi (KPa), values determined for the material constants are K = 5.48E−16 (3.18E−19), a = −0.654, and b = 1.43. These are the values used in representing the time-dependent behavior of rock salt for the present study. In this study also, shale is assumed to behave in the same manner as salt, but to be twice as viscous as salt under the same conditions. Both salt and shale are assumed incompressible.

FINITE ELEMENT METHOD

The finite element method (FEM) provides a convenient means of handling both the model geometry and material representation. Figure 3 shows the cross-sectional area being studied. The area is subdivided into a typical FEM mesh composed of linear isoparametric quadrilateral elements. Element definitions respect the geometry of the dome flank. Pressure boundary conditions can be applied within the cavern, and along the left, upper and right boundary, as seen in Figure 3. Element nodes lying on the lower boundary, across which symmetry is implied, are constrained to remain on the boundary and the lower right node of the model is fixed to exclude the possibility of rigid body motion. Effective viscosity is assumed constant over each element and is evaluated at each element mid-point. The FEM formulation employed uses a penalty function (Zienciewicz, 1974; Hughes, *et al.*, 1979) technique to enforce approximate incompressibility on a per element basis.

Use of the FEM method reduces the analysis of the model to solve a system of non-linear equations which yield nodal velocities corresponding to a particular time t for a fixed geometry. Using matrix notation, the form of the system to be solved for a vector of nodal velocities (v) is:

$$[K](v) + (f) = \emptyset,\ K = K((v)). \tag{3}$$

The most straightforward means of resolving these equations is by direct iteration, reforming the matrix [K] (analogous to a stiffness matrix for elastic solids) for each iteration. Iteration is continued until a selected conver-

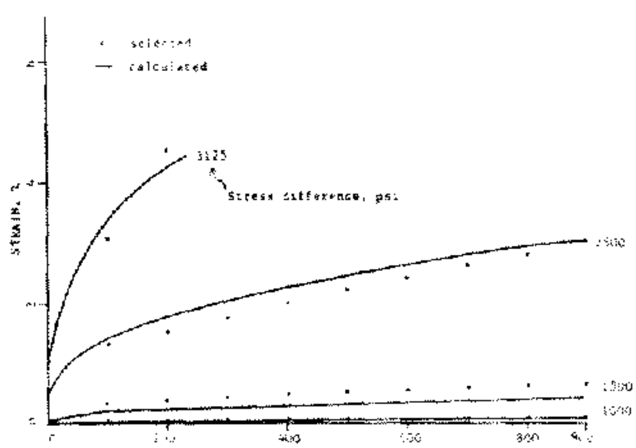

Figure 2. Triaxial Extension Creep Curves for Salt

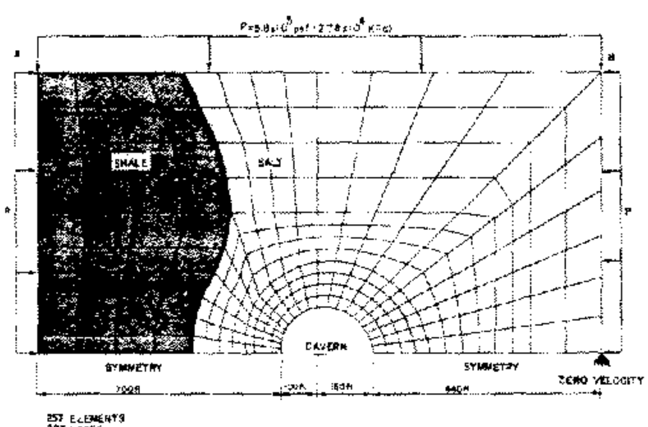

Figure 3. Finite Element Mesh and Boundary Conditions

gence criteria is satisfied. The convergence criteria used here is that the norm of the difference $|e|$ of two successive velocity vectors be less than some fraction 'c' of the norm of the latest velocity.

$$|e| < c|v|$$
$$|e| = \langle e \rangle (e),\ (e) = (v)_i - (v)_{i-1} \qquad (4)$$

Solution for deformation of the model over time is performed by use of a simple time-marching scheme with suitably small time steps and geometry update.

RESULTS

Boundary conditions for this analysis were uniform lithostatic pressure (5.8E5 psf, 27.8 MPa) applied along the outer model edges and zero cavern pressure assuming an empty cavern. These conditions were not changed over the duration of the analysis.

The iterative solution process was started for time $t = 1$ hour and the initial geometry of Figure 3 with an initial estimate obtained from a linear elastic analysis. Salt was assumed to be twice as stiff as shale and a Poisson's ratio of 0.30 was employed.

The solution for time $t = 1$ hour converged after only 12 iterations with a convergence criteria of $c = .001$. Figure 4 shows the stress distribution obtained for a time $t = 720$ hours (approximately 1 month). Negligible difference was seen between the stress distribution at $t = 1$ hour and $t = 720$ hours. No observed effect is attributable to the presence of the dome flank.

Figure 5 is a plot of the square root of the second invariant of deviatoric stress. The model was completed across the boundary with symmetry for plotting this surface. Again, no significant variation is apparent due to the location of the dome flank. Loss of cavern cross-sectional area over time is depicted in Figure 6.

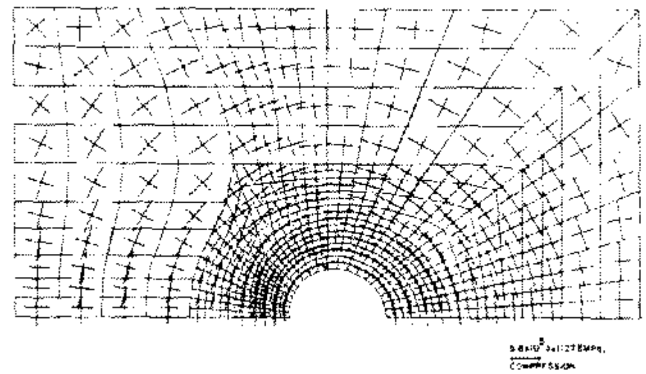

Figure 4. Distribution of Principal Stress for Viscous Flow, $t = 720$ hours

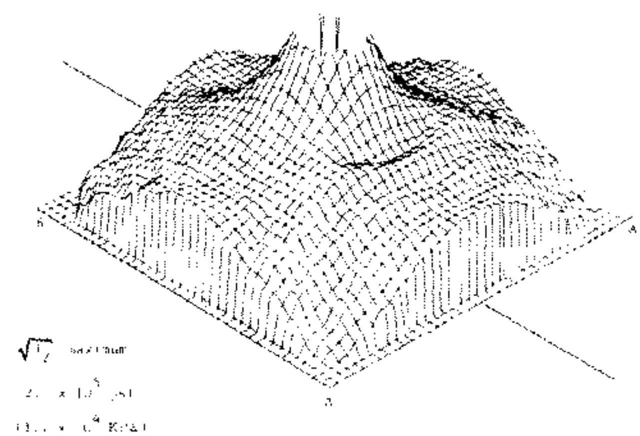

Figure 5. Surface Defined by $\sqrt{J_2}$ for Viscous Flow, $t = 720$ hours

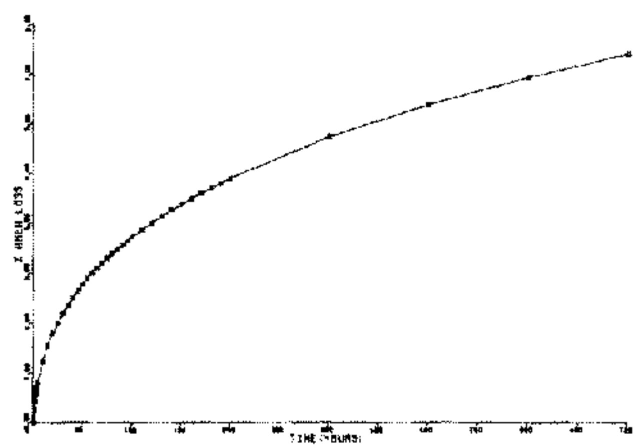

Figure 6. Percentage Loss in Cavern Cross-Sectional Area Versus Time

CONCLUSION

Obviously more work needs to be done. The model performed admirably, yielding an extremely stable, albeit uninteresting, solution. The most obvious step to take in pursuing this analysis would be to use a more realistic material description to represent the outlying material. Also of interest would be a sensitivity analysis associated with the proximity of the dome flank to the cavern. A study including different cavern shapes and various cavern load conditions, as has been done (Ghaboussi, *et al.*, 1981) for caverns well away from the dome boundary, is also of interest for near-flank caverns.

It should be noted that while nothing dramatic in cavern behavior is predicted for the specific situation considered, a cavern located near the flank of a salt dome poses problems beyond the question of structural stability. If a cavern near the flank of a dome is used for storage purposes, the question of degree of containment of the stored material takes on added significance.

Hydraulic conductivity of rock salt is affected by its degree of confinement and the associated stress field. A salt-shale interface in the proximity of a storage cavern implies a need for additional considerations of stress field variations on the surrounding host rock system. Increases in host rock hydraulic conductivity may range from slow "leaks" to more abrupt "hydraulic fracing." The latter case involves a phenomenon that is site specific, and which has not been introduced in this preliminary paper. However, it is generally acknowledged that stress states considered acceptable for interior caverns may be unacceptable for caverns near dome boundaries.

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